STRONG SPLIT LICT DOMINATION OF A GRAPH

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ABSTRACT

In this paper we initiate the study of a variation of standard domination such as strong split Lict domination. A dominating set D_{ss} of a lict graph $n(G)$ is a *strong split lict dominating set* if $\langle V(n(G)) - D_{ss}\rangle$ is totally disconnected with at least two vertices. The *strong split lict domination number* of a graph is the minimum cardinality of the strong split lict dominating set of G and is denoted by $\gamma_{ssn}(G)$. In this paper γ_{ssn}- number of some standard graphs is obtained. Also we established upper bounds and lower bounds on γ_{ssn}- number in terms of elements of G.

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1. Introduction: The graph theoretical terminology not present here can be found in Harary [2].All the graphs considered here are simple, finite, non-trivial, undirected and connected. As usual $p=|V|$ and $q=|E|$ denote the number of vertices and edges of a graph G. The study of domination in graphs was begun by Ore [8] and Berge [1].The domination in graphs is discussed by T.W.Hynes, S.T.Hedetniemi and P.J. Slater in [6]

For any graph $G=(V,E)$,the *lict graph* $n(G)$ has vertex set as the union of the set of edges and the set of cut vertices of graph G in which two vertices of $n(G)$ are adjacent if and only if their corresponding elements in G are adjacent or incident. This concept was introduced in [5].

We begin by recalling some standard definition from domination theory.

A set D $\subseteq V(G)$ is said to be a *dominating set* of G, if every vertex in $(V-D)$ is adjacent to at least one vertex in D. The minimum cardinality of minimal dominating set D is called *domination number* of G and is denoted by $\gamma(G)$.

Edge dominating set $F \subseteq E(G)$ is such that every edge in $(E(G)-F)$ must be adjacent to at least one edge in F. The minimum cardinality of edge dominating set is called *edge domination number* and is denoted by $\gamma'(G)$. Edge domination number was studied by S.L. Mitchell and Hedetniemi in [7].

A set D_ε of elements of G is an *entire dominating set* if every element not in D_ε is either adjacent or incident to at least one element in D_ε. The *entire domination number* is denoted as $\gamma_{\varepsilon n}(G)$. This concept was introduced in [4].

A dominating set D_s of G is called *strong split dominating set* of G if $\langle V(G)-D_s\rangle$ is totally disconnected with at least two isolated vertices. The *strong split domination number* $\gamma_{ss}(G)$ is the minimum cardinality of minimal strong split dominating set. This concept of strong split domination was introduced in [3].

Analogously we define *strong split lict dominating set* of a graph G. A set D_{ss} is said to be strong split lict dominating set if $\langle V(n(G))-D_{ss}\rangle$ is totally disconnected with at least two vertices. The *strong split lict domination number* of a graph G is the minimum cardinality of the strong split lict dominating set of G and is denoted by $\gamma_{ssn}(G)$.

A vertex cover in a graph G is a set of vertices that covers all the edges of G. The *vertex covering number* $\alpha_0(G)$ is a minimum cardinality of a vertex cover in G. An edge cover of a graph G without isolated vertices is a set of edges of G that covers all the vertices of G. The *edge covering number* $\alpha_1(G)$ of a graph G is the minimum cardinality of an edge cover of G. A set of vertices/edges in a graph G is called an *independent set* if no two vertices/edges in the set are adjacent. The *vertex independence number* $\beta_0(G)$ is the maximum cardinality of an independent set of vertices. The *edge independence number* $\beta_1(G)$ of a graph G is the maximum cardinality of an independent set of edges.

A set of vertices S in a graph G is called an *independent set* if no two vertices in S are adjacent. The *lower independence number* $i(G)$ is the minimum cardinality of a maximal independent set of G.

The minimum distance between any two farthest vertices of a connected graph G is called the *diameter* of G and is denoted by $diam(G)$.

The *clique number* of a graph G is the maximum order of the largest clique in G and it is denoted as $\omega(G)$.

We need the following Theorems to establish our further results:

Theorem A [7]: If G is a graph with no isolated vertex, then $\gamma(G) \leq \frac{p}{2}$

Theorem B [3]: For any connected graph G,$(G) \neq K_p, \gamma_{ss}(G) = \alpha_0(G)$.

In section 2, we determine this parameter for some standard graphs .We obtain best possible upper and lower bound for $\gamma_{ssn}(G)$.Further we obtain Northus Gaddum type results.

2. RESULTS

First we list out the exact values of $\gamma_{ssn}(G)$ for some standard graphs.

Theorem 1:

1) For any cycle C_p with $p \geq 4$ vertices $\gamma_{ssn}[C_p] = \begin{cases} \frac{p}{2} \; if \; p \equiv 0(mod\ 2) \\ \left\lceil \frac{p}{2} \right\rceil \; otherwise \end{cases}$

2) For any path P_p with $p \geq 4$ vertices $\gamma_{ssn}[P_p] = p - 1 = q$.

3) For any complete graph K_p with $p \geq 4$ vertices $\gamma_{ssn}[P_p] = \frac{p(p-1)}{2} - 2$.

4) For any bipartite graph $K_{m,n}$ with $m \leq n$, $\gamma_{ssn}[K_{m,n}] = m.n - m$.

The following are some observations on $\gamma_{ssn}[G]$:

i) For any graph $G = K_{m,n}; m, n \geq 2, m < n$,

$n(K_{m,n}) = K_m \times K_n$.

Hence $\gamma_{ssn}(K_m \times K_n) = m.n - m$.

ii) For any graph G , $\gamma_{ss}(G) \leq \gamma_{ssn}(G)$. Equality holds for any $G = C_p$.

iii) For any path P_p :

a) $\gamma_{ssn}[P_p] = \gamma_{ss}[L(P)_p] + \left\lfloor\frac{p}{2}\right\rfloor$ where $p \geq 4$.

b) $\gamma_{ssn}[P_p] = \gamma_{ssn}[L(P)_p] + 1$ where $p \geq 5$.

In the following Theorem we established the equality for γ_{ssn} of a wheel.

Theorem 2: For a wheel Wp; p ≥ 4 vertices

$$\gamma_{ssn}(W_p) = \begin{cases} p+n, & n = \frac{p-4}{2}, \quad P = \text{even} \\ p+m, & m = \frac{p-3}{2}, \quad P = \text{odd} \end{cases}$$

Proof: Let V[Wp] = $\{v_1, v_2, \ldots\ldots v_p\}$; deg $v_i = 3$, for i = 1, 2, ……p – 1 and deg $v_p = p - 1$. Let E $[W_p] = \{e_1, e_2, \ldots\ldots, e_{p-2}, e_{p-1}, e'_1, e'_2 \ldots\ldots e'_{p-1}\}$ where each $e_i = v_i v_p$ With i = 1, 2, …….p– 1 and $e'_i = v_i v_{i+1}$, i = 1,2,……p-2 and $e'_{p-1} = v_{p-1} v_1$. Now V $[n(W_p)] = \{e_1, e_2, \ldots\ldots, e_{p-2}, e'_1, e'_2 \ldots\ldots e'_{p-1}\}$. Consider $S_1, S_2 \subseteq$ V [*n* (Wp)], such that $S_1 = \{e_1, e_2, \ldots\ldots e_{p-2}, e_{p-1}\}$and $S_2 = \{e'_1, e'_2, \ldots\ldots e'_{p-2}, e'_{p-1}\}$.The induced subgraph $\langle S_1 \rangle$ is a complete subgraph K_{P-1} and induced subgraph $\langle S_2 \rangle$ is a cycle in *n* (G). We consider the following two cases.

Case 1: Suppose p is even. Then $|S_2|$ in an odd number, consider $S'_1 \subseteq S_1$ with $S'_1 = \{e_1, e_2, \ldots\ldots e_{p-2}\}$ and consider another set $S'_2 \subseteq S_2$ such that

$S'_2 = \{e'_2, e'_4, \ldots\ldots, e'_{P-4}, e'_{p-2}, e'_{p-1}\}$.Clearly $\langle V(n(W_p) - (S'_1 \cup S'_2) \rangle$ is totally disconnected. Then $(S'_1 \cup S'_2)$ is a strong split dominating set of n(Wp) by the definition.

Hence $\gamma_{ssn}(W_p) = |S'_1 \cup S'_2| = (P-2) + 2 + \frac{P-4}{2}$

Which results in to γ_{ssn} (Wp) = P + n, where $n = \frac{P-4}{2}$

Case 2: Suppose p is odd. Then $|S_2|$ = even number. We consider $S''_1 \subseteq S_1$, with $S''_1 = \{e_1, e_2, \ldots\ldots e_{P-2}\}$ and another set $S''_2 \subseteq S_2$, such that $S''_2 = \{e'_2, e'_3, e'_5, e'_7, e'_9 \ldots\ldots e'_{P-5}, e'_{P-3} e'_{P-2}, e'_{p-1}\}$. Clearly $S''_1 \cup S''_2$ forms a strong split lict dominating set of W_p, since $\langle V[n(W_p)] - (S''_1 \cup S''_2) \rangle$ is totally disconnected.

Then $\gamma_{ssn}(W_p) = |(S''_1 \cup S''_2)|$, also S''_1, S''_2 are vertex disjoint sets, so it results in to $\gamma_{ssn}(W_p) = p + \frac{p-3}{2} = p+m$, where $m = \frac{p-3}{2}$, p = odd.

Here we have found an upper bound for γ_{ssn} (G) in terms of γ(G).

Theorem 3: For any connected (p.q) graph G, $n(G) \neq K_p$ with $p \geq 4$ vertices, $\gamma_{ssn}[G] \geq p - [\gamma(G)+1]$. Equality holds for C_{2p}, $p \geq 3$.

Proof: In n (G), V [$n(G)$] = E (G) $\cup$ C (G). If $n(G) = K_p$, Then by definition, γ_{ssn} - set does not exist. We consider the following two cases.

Case 1: Suppose G is a tree. Then in $n(G)$each block is complete. Let S = $\{e_1, e_2, \ldots\ldots e_i\}$ be the set of all nonend edges in G and each $e_i \in G$ preserves a one to one correspondence with $v_i \in D$, where $D = \{v_1, v_2, v_3 \ldots . v_i\}$ and $D \subseteq V[n(G)]$. Suppose |D| = h.Then $\langle V[n(G)] - D \rangle$ = H.Also H has maximum h+1 components. Now we partition H into H_1 and H_2, where $H_1 = \{v_j\}$. $1 \leq j \leq h+1$, and each v_j is an isolate in H. Consider $H_2 = \{b_k\}$. $1 \le k \le h+1$ and each b_k is a complete block with m vertices; $m \ge 2$. We consider a dominating set D_1 which is not minimal in H_2. The set D_1 consists of (m-1) vertices from each block b_k, such that $\langle V[n(G)] - (D \cup D_1)\rangle$ is totally disconnected, it implies that $(D \cup D_1)$ is γ_{ssn} - set of G.

Further let D is the minimal dominating set of G, then at least one vertex $\nu \in V[n(G)]$ such that $\nu \in (D_1 \cup D_2) \cap D$ then it follows

$|D_1 + D_2|+|D| +1 \ge p$ resulting into $\gamma_{ssn}(G) = p - [\gamma(G) +1]$.

Case 2: Suppose G is not a tree. Then G has $p \ge 4$ and it contains at least one block with minimum three vertices.Now $V[n(G)] = E(G) \cup C(G)$. Let $S_1 = \{e_1, e_2, \ldots\ldots e_j\}$ be the set of nonendbridges of G, which preserves one to one correspondence with the vertex set $D_1 \subseteq V[n(G)]$. Let $|D_1| = h$, then $\langle V[n(G)] - D_1\rangle = H$ has h+1 components. We consider $H = H_1 \cup H_2 \cup H_3$; where $H_1 = \{\nu_i\}$ where each $\nu_i \in V[(n(G)]$ is an isolate and H_2 is a set of vertex disjoint components which are complete, $H_2 = \bigcup_{i=1}^{h+1} K_{m_i}$, where each K_{m_i} is a complete block with m_i vertices, $m_i \ge 2, 3, 4\ldots\ldots$ for $i \ge 1$ and $H_3 = \bigcup_{j=1}^{h+1} B_j$ where each B_j is a noncomplete block. We consider $D_2 = \{\nu_i\} \subseteq V[H_2]$ which includes $(m_i - 1)$ vertices from each $K_{m_i} \in H_2$ gives an isolate, then $(H_2 - D_2)$ is totally disconnected, Further we consider $D_3 = \{\nu_1, \nu_2\ldots\ldots\nu_k\}$ $\forall\, \nu_i \in Bj$, $1 \le i \le k$, and degree of each ν_i is $\delta(B_j)$

Now we consider $B_j - D_3 = H_4$ in which each element is an isolate, then $(D_1 \cup D_2 \cup D_3)$ is such that $\langle V[n(G)] - (D_1 \cup D_2 \cup D_3)\rangle$ is totally disconnected. So $(D_1 \cup D_2 \cup D_3)$ is a γ_{ssn} - set. And if D is the minimal dominating set of G then there exist at least one vertex $\nu_i \in C(G)$ such that $\nu_i \in D \cap (D_1 \cup D_2 \cup D_3)$ then it gives $|D_1 \cup D_2 \cup D_3| + |D| + 1 \ge p$ or $(D_1 \cup D_2 \cup D_3) \ge p - (|D|+1)$ such that $\gamma_{ssn}(G) \ge p - (\gamma(G) + 1)$.Hence the result.

For equality:Let C_p be a cycle with $p \ge 6$ or $G = C_{2n}$, $n \ge 3$.

Let D_{ss} be the minimal strong split lict dominating set of G, then

$|D_{ss}| = \left\lceil \frac{p}{2} \right\rceil$; whereas $p - [\gamma(G) + 1] = p - \left(\left\lceil \frac{p}{3} \right\rceil + 1\right)$. Now one can easily verify that $|D_{ss}| = p - (\gamma(G) + 1)$ for a cycle C_{2n}, $n \geq 3$.

Theorem 4: For any connected (p, q) graph G, with $p \geq 4$ vertices and $n(G) \neq K_p$, then

$$\gamma_\varepsilon [G] \leq \gamma(G) + \gamma'[G] \leq \gamma_{ssn}[G] .$$

Proof: First we establish the lower bound. Let D and F be the minimal dominating set and edge dominating set of G. Then $D \cup F$ is an entire dominating set of G. Thus

$$\gamma_\varepsilon (G) \leq |D \cup F| = \gamma(G) + \gamma'(G).$$

For the upper bound Let D_{ss} is a γ_{ssn} - set of G and let $F_1 = \{e_1, e_2, \ldots\ldots\ldots e_n\}$ be the minimal edge dominating set of G, then there exist $D_1 = \{ \nu_1, \nu_2, \ldots\ldots\ldots \nu_n\} \subseteq V[n(G)]$ $\forall$ $\nu_i \in D_1$, ν_i corresponds to $e_i \in F_1$, such that,$D_1 \subseteq D_{ss}$. We take $F_2 = \{e_j\}$ where each e_j is nonendbridge in G, then $D_2 = \{\nu_j\}$ is a set of all cut vertices in n(G) such that each $\nu_j \in D_2$ corresponds to $e_j \in F_2$ and $D_2 \subseteq D_{ss}$. Now consider a set $V' \subseteq V [n(G)]$ such that $V' \in N (D_1 \cup D_2)$, so that $V' \subseteq V[n(G)] - (D_1 \cup D_2)$ and $\langle V [n(G)] - (V' \cup D_1 \cup D_2)\rangle$ is totally disconnected then

$$(D_1 \cup D_2) \cup V' = D_{ss} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

Now let D_3 is the minimal dominating set of G, then D_3 consists of vertices which are incident to at least one edge $e_j \in F_2$. Also in G each cut vertex $\nu_j \in D_3$ is such that $\nu_j \in V' \subseteq V [n (G)]$, then $|D_3| \leq |F_2| + |V'|$ which implies

$$|D_3| \leq |D_2| + |V'| \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (2)$$

Combining (1) and (2) we get $|D_1| + |D_3| \leq |D_{ss}|$ which gives

$$\gamma(G) + \gamma'(G) \leq \gamma_{ssn} (G).$$

Thus $\gamma_{\varepsilon n} (G) \leq \gamma(G) + \gamma'(G) \leq \gamma_{ssn} (G)$.

Equality for strong split domination number of a tree is established in the following Theorem.

Theorem 5: For any connected (p, q) tree T, with $p \geq 4$ vertices, $T \neq K_{1,n}$ with $n \geq 2$, then $\gamma_{ssn}(T)=q$

Proof: Suppose T is a tree and assume $T = K_{1,n}$. Then $n(T) = K_p$, from the definition of strong split lict domination, γ_{ssn} - set does not exist.

Further for any tree T, with $p \geq 4$, each block in n (T) is complete and each cut vertex of n(G) lies on exactly two blocks. Now $V[n(G)] = E(G) \cup C(G)$ and $V[n(G)] - E(G) = C(G)$ gives a disconnected graph such that $\forall\, v_i \in C(G)$ is an isolate. Clearly E(G) is a Y_{ssn}-set of T. Hence $|E(G)| = q = Y_{ssn}(G)$.

Theorem 6: If G is a connected graph of order $p \geq 4$ and $n(G) \neq K_p$, then

$$\left\lceil \frac{diam G+1}{2} \right\rceil \leq \gamma_{ssn}(G).$$ Equality holds for C_4.

Proof: Suppose $E(G) = \{e_1, e_2, \ldots..e_n\}$ and $C(G) = \{c_1, c_2, c_3, \ldots..c_j\}$ be the edge set and cutvertex set of G respectively. Then $V[n(G)] = E(G) \cup C(G)$. Let $S = \{e_1, e_2, \ldots..e_j\}$,$1 \leq j \leq n$ constitute the diameteral path in G. Then $|S| = diam\ G$. Let D be a dominating set in n(G) and $D_1 \subseteq V[n(G)] - D$, such that $D_1 \in N(D)$, again we take $D'_1 \subseteq D_1$ such that $H = V[n(G)] - (D \cup D'_1)$ and $\langle H \rangle$ is totally disconnected. Hence $D \cup D'_1 = D_{ss}$. Further since $S \subseteq V[(n(G)]$ and $D \cup D'_1$ is a γ_{ssn} - set, the diameteral path includes at most $\gamma_{ssn}(G) - 1$ vertices which belongs to neighborhood of $(D \cup D'_1)$ in $n(G)$. Hence $diam\ G \leq \gamma_{ssn}(G) + \gamma_{ssn}(G) - 1$ which follows $\left\lceil \frac{diam G+1}{2} \right\rceil \leq \gamma_{ssn}(G)$.

One can easily verify for the equality.

Theorem 7: For any connected (p, q) graph G, $n(G) \neq K_p$ then ,$\gamma_{ssn}(G) \leq \alpha_1(G) + \alpha_1[n(G)] - 1$ Equality holds for Wp, $p \geq 4$ vertices.

Proof: Consider a set $D = \{e_i\}$, $i = 1, 2 \ldots. n$ be the maximal edge cover of G, so that

$|D| = \alpha_1$ (G). Further V [n (G)] = {e_j}, where each $e_j = e_i e_j$ or $e_j c_k$ where adjacency of e_i with e_j, and adjacency of e_j with cut vertex c_k is preserved as in G. We take a spanning tree H of n (G) and let $D_1 = \{e'_1, e'_2, ...e'_k\} \subset E$ [n (G)] be the edge cover of H, then $|D_1| = \alpha_1 (H) = \alpha_1 [n (G)]$. Consider $H_1 = \{e''_j\}$ where each e''_j is only one edge chosen among all edges $e_i e_j$ incident to vertex e_j in n (G) which corresponds to edge e_j of G and $e_j \in \alpha_1$(G) set, then $H_1 \subset \alpha_1$ (G)-set. Further we consider a strong split dominating set $D_{ss} = \{e_j, c_k\}$ in n (G), such that $D_{ss} \subseteq H \cup H_1$, which gives $D_{ss} \subseteq |D| + |D_1| - 1$ resulting in to $\gamma_{ssn} (G) \leq \alpha_1 (G) + \alpha_1 (n (G)) - 1$.

Theorem 8: For any connected (p, q) graph G. n (G) $\neq K_p$, p $\geq$ 4vertices, $\gamma_{ssn} (G) \geq \left\lceil \frac{p}{2} \right\rceil$

Proof: We consider the following cases:

Case 1: Suppose G is a tree, with H = {$v_1, v_2 \ldots v_i$} be the set of all vertices in T .Then I = [$v_1, v_2, \ldots. v_j$] be the set of all end vertices in T and let H′ = [$e_1, e_2, \ldots$ei] be the set of all non end edges in T, also I′ = {$e_1, e_2, \ldots..e_j$} be the set of all end edges in T. In n (T), C be the set of all cut vertices in T, V [n (T)] = $H' \cup I' \cup C$. Suppose D_{ss} be a γ_{ssn} - set of T such that $D_{ss} = H' \cup I''$ where $I'' \subseteq I'$, which gives

$$|H' \cup I''| = \gamma_{ssn} (T) \geq \frac{H \cup I}{2} \text{ resulting in to } \gamma_{ssn} (G) \geq \left\lceil \frac{p}{2} \right\rceil.$$

Case 2: Suppose G is not a tree, then there exists at least one edge joining two distinct vertices of a tree T, which forms a cycle. From the above case 1

$|V (n (G))| \geq |H' \cup I' \cup C| + 1$ which gives

$$|H' \cup I''| + 1 \geq \left| \frac{H \cup I}{2} \right| + 1 \text{ resulting in to } \gamma_{ssn} (G) \geq \left\lceil \frac{p}{2} \right\rceil.$$

Theorem 9: For any connected (p, q) graph G with n (G) $\neq K_p$, p $\geq$ 4 vertices, $\gamma_{ssn}(G) \geq \gamma(G)$. Equality holds if G = C_4.

Proof: Let G be a connected (p, q) graph and D_{ss} and D are γ_{ssn} - set and γ-set of G respectively. By the Theorem [A] and Theorem [8], we have $\gamma_{ssn}(G) \geq \gamma(G)$. Also one can easily verify the equality.

Theorem10: For any connected (p, q) graph G. $n(G) \neq K_p$, $p \geq 4$vertices ,$\gamma_{ss}[L(G)] \leq \gamma_{ssn}(G)$ equality holds if G is a block graph.

Proof: Since $V[n(G)] \supseteq V[L(G)]$ by definition. Hence the result follows. And if G = block graph then $V[n(G)] = V[L(G)]$ which gives the equality.

Theorem 11: For any connected (p, q) graph G, with $p \geq 4$ vertices, $n(G) \neq K_p$,

$\gamma_n(G) + \gamma_{ssn}(G) \leq q + c$,where c is the number of cutvertices in G.

Proof: Suppose G has $p \leq 3$.Then γ_{ssn} - set does not exist. Now we consider any graph with $p \geq 4$, such that $n(G) \neq K_p$

Since , $\gamma_n(G) \leq \beta_0[n(G)]$ and from Theorem B

$$\gamma_{ssn}[G] = \alpha_0[n(G)].$$

Further $\gamma(G) + \gamma_{ssn}(G) \leq \alpha_0[n(G)] + \beta_0[n(G)]$

$= V[n(G)]$

$= q + c$

Hence $\gamma(G) + \gamma_{ssn}(G) \leq q + c$.

Theorem 12: For a connected (p, q) graph with $p \geq 4$ vertices and $n(G) \neq K_p$, then

$\gamma_{ssn}(G) \leq q + c - 2$, where c is the number of cutvertices in G.

Proof: It is known that

$\gamma_{ssn}[G] = \alpha_0[n(G)]$

$= V[n(G)] - \beta_0[n(G)]$.

Let $n(G) = H$, by Theorem B

$\gamma_{ssn}(H) = \alpha_0(H)$

Thus $\gamma_{ssn}[H] = V(H) - \beta_0(H)$

Further it is known that $\omega(\overline{H}) = \beta_0(H)$

Hence $\gamma_{ssn}(G) = V[n(G)] - \omega[\overline{n(G)}]$

Since , $\omega[\overline{(G)}] \geq 2$

Then $\gamma_{ssn}(G) \leq q + c - 2$.

Theorem 13: For any connected (p, q) graph G, with $p \geq 4$ vertices and $n(G) \neq K_p$, then

$i[n(G)] + \gamma_{ssn}[G] \leq q + c$, where c is the number of cutvertices in G.

Proof: Since $i[n(G)] \leq \beta_0 [n(G)]$

and $Y_{ssn}[G] \leq \alpha_0 [n(G)]$

$$i[n(G)] + \gamma_{ssn}[G] \leq \alpha_0 [n(G)] + \beta_0 [n(G)]$$

$$= V[n(G)]$$

$$= q + c.$$

Then $i[n(G)] + \gamma_{ssn}[G] \leq q + c$.

Theorem 14: For any connected (p, q) graph G with $p \geq 4$vertices and $n(G) \neq K_p$, then

$$\left\lceil \frac{p}{\Delta(G)+1} \right\rceil \leq \gamma_{ssn}(G)$$

Proof: Let D be a γ - set of G and each vertex dominates at most itself and $\Delta(G)$ other vertices, so $\left\lceil \frac{P}{\Delta(G)+1} \right\rceil \leq \gamma(G)$ and from Theorem [9]

$$\left\lceil \frac{P}{\Delta(G)+1} \right\rceil \leq \gamma_{ssn}(G)$$

Finally we obtain Northus Gaddum Type results.

Theorem 15: For any connected (p, q) graph G and $\overline{G}$ with $p \geq 4$ vertices and $n(G) \neq K_p$, then

(i) $\gamma_{ssn}(G) + \gamma_{ssn}(\bar{G}) \leq p+q$

(ii) $\gamma_{ssn}(G) \cdot \gamma_{ssn}(\bar{G}) \leq \left\lceil \frac{p.q}{2} \right\rceil$

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